ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 3, Summer 1984

UPPER AND LOWER SOLUTIONS FOR SYSTEMS OF SECOND ORDER EQUATIONS WITH NONNEGATIVE CHARACTERISTIC FORM AND DISCONTINUOUS NONLINEARITIES

CHRIS COSNER AND FRANK SCHINDLER

1. Introduction. The method of upper and lower solutions has a long history and a wide application in the study of nonlinear elliptic and parabolic boundry value problems. It is related to the methods of monotone operators as discussed in [1], and has been applied to elliptic, parabolic, and first order equations and systems by various authors, see [1], [2], [3], [4], [8], [10], [11], [12], [13], among many others; for further references see [1], [12].

The present article gives a unified treatment for systems of second order equations with nonnegative characteristic form, as studied in [5], [6], [9]. Such equations include elliptic, parabolic, and first order equations, among others. The results obtained here extend existing ones in various ways. First, we consider the general setting of systems of equations with nonnegative characteristic form. Single equations of that type are considered in [4], but are required to be elliptic near the boundary of the domain where they are studied. We consider weak solutions; in fact, since we allow discontinuities in our nonlinearity, and equations with nonnegative characteristic form need not have any smoothing properties, we can do no better.

Specifically, we consider systems of the form

$$-L^r[u^r] = f^r(x, \vec{u}) \text{ in } \Omega, \qquad r = 1, \ldots, m \tag{1.1}$$

where $\Omega \subseteqq \mathbf{R}$ is a smooth bounded domain, $\vec{u} = (u^1, \ldots, u^m)$ and

$$L^r[u] \equiv \sum_{i,j=1}^{n} a_{ij}^r(x) u_{x_i x_j} + \sum_{i=1}^{n} b_i^r(x) u_{x_i} + c^r(x) u \tag{1.2}$$

with $\sum_{i,j,=1}^{n} a_{ij}^r(x)\, \xi_i \xi_j \geqq 0$. The functions $f^r(x, \vec{u})$ are required to be measurable in all variables and quasimonotone, that is, f^r must be nondecreasing in u^s for $s \neq r$; we also require that $f^r(x, \vec{u}) + Mu^r$ be increasing in u^r for some $M > 0$. We also require that if $\vec{u}$ is bounded and

Received by the editors June 24, 1983.