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MINIMAL EXISTENCE OF NONOSCILLATORY SOLUTIONS IN FUNCTIONAL DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENTS

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ABSTRACT. For the equation

(A) $$L_n y(t) + F(t, y(g(t))) = f(t)$$

minimal sufficient conditions ensure the existence of a nonoscillatory solution of (A). L_n is a disconjugate differential operator of the form

$$L_n = \frac{1}{P_n(t)} \frac{d}{dt} \frac{1}{p_{n-1}(t)} \cdots \frac{1}{p_1(t)} \frac{d}{dt} \frac{\cdot}{p_0(t)}.$$

1. Introduction. It is well known from the works of Onose [3] and Singh [7] that, subject to the conditions

(1) $$\int^{\infty} t^{n-1}|q(t)|\, dt < \infty$$

and

(2) $$\int^{\infty} t^{n-1}|f(t)| dt < \infty,$$

an equation of the form

(3) $$y^{(n)}(t) + q(t)y(g(t)) = f(t)$$

has a nonoscillatory solution with a prescribed limit at ∞. However when the integral size in (1) or (2) is allowed to be unbounded, then the results of Singh [7], Onose [3], Lovelady [2] and Philos [4] do not indicate if a nonoscillatory solution still exists. Our purpose in this work is to prove the existence of a nonoscillatory solution of a much more general functional equation of the form

(4) $$L_n y(t) + F(t, y(g(t))) = f(t)$$

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