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SELECTIONS OF MULTIFUNCTIONS OF TWO VARIABLES

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0. Introduction. Let T, X, Y be three non-empty sets and F be a multifunction from $T \times X$ into Y. Then we may ask whether there exists a selection of F having a certain type of regularity with respect to the first variable and a certain other with respect to the second one. In practice, one provides the sets T, X, Y with some measurable or topological or even algebraic structures to consider types of regularity as measurability, continuity, belonging to some Baire class and so on. In particular, it is interesting to find Carathéodory's selections. Recently, in [1], we have given a contribution to this problem.

The aim of the present paper is to show how the technique of proof used in [1] can be formalized in such a manner as to obtain, in a unified way, several results about our present and more general problems. Thus, as corollaries of a unique abstract theorem (Theorem 2.1), two specified versions of the results of [1] are obtained as well as improvements in several directions of some remarkable particular cases of the results contained in [2] and [3]. We want also to stress that some theorems we derive from Theorem 2.1 provide, as their consequences, new results on selections of class α on $T \times X$ ($0 \leqq \alpha < \omega_1$) of the given multifunction F, in the case where T, X, Y are topologized. In particular, if $\alpha = 0$, we don't assume that the product space $T \times X$ is normal.

The present paper has four sections. §1 is devoted to the notations used and to the definitions which are needed in order to state Theorem 2.1. This result is proved in §2. In §3 we explain the definitions put in §1. Finally, in §4 we present several consequences of Theorem 2.1 which differ from one another in some remarkable feature of topological nature. We conclude by establishing, as an application of a result previously obtained, an existence theorem on differential inclusions in Banach spaces which extends Theorem 2 of [3].

1. Notations and definitions. For every set $A \neq \emptyset$, we denote by $\mathscr{P}(A)$ the family of all subsets of A and by 2^A the family $\mathscr{P}(A)\backslash\{\emptyset\}$. Given two sets A', $A'' \neq \emptyset$, we denote by $\mathscr{F}(A', A'')$ the set of all functions from A' into A''. Given two sets B, $C \neq \emptyset$ and given $F \in \mathscr{F}(B, 2^C)$ (resp.,

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