A CHARACTERIZATION OF RATIONAL AND ELLIPTIC REAL ALGEBRAIC CURVES IN TERMS OF THEIR SPACE OF ORDERINGS*

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1. Introduction. D.O.P. fields were introduced by Dubois and Recio in [3] as a class of fields whose orderings are all "almost" isomorphic (see 2.1 for a precise definition). There they prove that finitely generated pure transcendental extensions of real closed fields are D.O.P.

In a more general setting, we study here under what conditions the field of rational functions of an algebraic variety V of $\mathbf{R}^n$ is D.O.P. We obtain a necessary condition for arbitrary dimension (prop. 2.4) and a necessary and sufficient condition for curves (theorem 3.1).

The D.O.P. property turns out to be closely related to the automorphism group of the field of rational functions of the variety and consequently to Klein's surfaces and N.E.C. groups (see [2] and [5] for the basic definitions).

It will be seen in the proof of (3.2.1) that given two points of a real elliptic curve C there exists a birational morphism of C sending one of them to the other. This is a well known result if the ground field is algebraically closed (see [4]), but it is an open problem (as far as I know) working over real closed ground fields (different from $\mathbf{R}$).

2. Preliminaries and necessary condition. First we introduce some notation. Let K be a formally real field. We let Ω be the order space of K endowed with Harrison's topology. Let G stand for the automorphism group of K, and $x_1, \ldots, x_n$ for indeterminates.

DEFINITION 2.1. A formally real field K is D.O.P. (or has the dense orbits property) if for each ordering $\alpha \in \Omega$ and each open $H \subset \Omega$ there exists an automorphism $\sigma \in G$ such that $\sigma(\alpha) \in H$.

From [3] we need the following result.

PROPOSITION 2.2. *If R is a real closed field, then $R(x_1, \ldots, x_n)$ is D.O.P.*

On the other hand, in dealing with algebraic varieties we shall consider

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