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EXISTENCE AND UNIQUENESS OF SOLUTIONS OF RIGHT FOCAL POINT BOUNDARY VALUE PROBLEMS FOR THIRD AND FOURTH ORDER EQUATIONS

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1. Introduction. In this paper we consider the existence and uniqueness of solutions of right focal point boundary value problems for third and fourth order equations. To relate this to earlier results concerning right focal point boundary value problems, we shall first formulate such problems for equations of arbitrary order; thus, we shall be concerned with solutions of the equation

$$(1) \qquad y^{(n)} = f(x, y, y', \ldots, y^{(n-1)}),$$

satisfying boundary conditions of the form $y^{(i-1)}(x_i) = y_i$, $1 \leqq i \leqq n$, where $a < x_1 \leqq x_2 \leqq \cdots \leqq x_n < b$. Such a problem is called a right focal point boundary value problem for (1) on (a, b). To be more precise, we give the definition, as it appears in [6, 7], of a right $(m_1, \ldots, m_r)$-focal point boundary value problem for (1) on (a, b).

DEFINITION. Let $2 \leqq r \leqq n$ and let m_i, $1 \leqq i \leqq r$, be positive integers such that $\sum_{i=1}^{r} m_i = n$. Let $s_0 = 0$ and for $1 \leqq k \leqq r$, let $s_k = \sum_{i=1}^{k} m_i$. A boundary value problem for (1) with boundary conditions

$$y^{(i)}(x_k) = y_{ik},\ s_{k-1} \leqq i \leqq s_k - 1,\ 1 \leqq k \leqq r,$$

where $a < x_1 < \cdots < x_r < b$, is called a right $(m_1, \ldots, m_r)$-focal point boundary value problem for (1) on (a, b).

In addition to [6, 7], for results related to this type of boundary value problem, see for example Muldowney [13, 14], Nehari [15], Elias [2, 3], Jackson [10], and Peterson [16. 17]. Commas appear in the notation $(m_1, \ldots, m_r)$, instead of semicolons, to distinguish this concept from a similar but different concept used by Peterson [18].

Now if (1) is linear, the uniqueness of solutions of a particular right $(m_1, \ldots, m_r)$-focal point boundary value problem implies the existence of solutions of the same type problem for any assignment of y_{ik}. In [7], a type of "uniqueness implies existence" result for right focal point bound-

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