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ISOLATORS IN SOLUBLE GROUPS OF FINITE RANK

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ABSTRACT. A group G is said to have the *isolator property* if for every subgroup H of G the set $\sqrt{H} = \{g \in G: h^n \in H$ for some $n \geqq 1\}$ is a subgroup. G is said to have the *strong isolator property* if in addition $|\sqrt{H}: H|$ is finite whenever $H/\bigcap_{g \in G} H^g$ is finitely generated. It is well known that locally nilpotent groups have the isolator property, that finitly generated nilpotent groups have the strong isolator property and that the infinite dihedral group D has neither. However D clearly has a subgroup of finite index with the strong isolator property.

Our purpose here is to show that for soluble groups and linear groups the isolator property is closely associated with the finiteness of the (Prüfer) rank, while the strong isolator property is closely associated with polycyclicity.

For subgroups $H \leqq K$ of a group G let $\sqrt[K]{H}$ denote the set $\{g \in K: g^n \in H$ for some $n \geqq 1\}$. If $K = G$ write $\sqrt{H}$ for $\sqrt[G]{H}$. The group G is said to have the *isolator property* if $\sqrt{H}$ is a subgroup of G for every subgroup H of G. If in addition $|\sqrt{H} : H|$ is finite for every H such that $H/\bigcap_{g \in G} H^g$ is finitely generated say that G has the *strong isolator property.* In his Edmonton lectures ([1]) P. Hall proved that locally nilpotent groups and finitely generated nilpotent groups have the isolator and the strong isolator property respectively. If G_0 is the infinite dihedral group and $H = \langle 1 \rangle$ then trivially $\sqrt{H}$ is not a subgroup. However the cyclic subgroup of G_0 of index 2 clearly does have the strong isolator property. For any class Σ of groups say that a group G is almost a Σ-group if it has a normal Σ-subgroup of finite index. Thus G_0 above almost has the strong isolator property. For soluble groups and for linear groups there is a close link between groups that almost have the isolator property and groups with finite (Prüfer) rank. Our main results are as follows.

THEOREM A. *Let G be a finitely generated soluble group. Then G is polycyclic if and only if G almost has the strong isolator property.*

THEOREM B. *A torsion-free soluble group of finite rank almost has the*

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