ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 2, Spring 1984

UNIFORM APPROXIMATION BY RANDOM FUNCTIONS

GARY F. ANDRUS and LEON BROWN

1. Introduction. In this paper we prove a number of stochastic analogues of deterministic uniform approximation theorems.

One possible approach to a stochastic analogue of a deterministic theorem is a careful examination of the parameters involved in the proof of the deterministic case. If one can show that these parameters can be chosen in a measurable way then the probabilistic version should follow with little difficulty. However, there are drawbacks to this line of attack. Firstly, with each new deterministic theorem comes a new set of parameters and their corresponding measurability problems; this is often a long and tedious process. Secondly, one is rarely able to prove much more than a direct analogue to the deterministic case; when in fact one has in mind more general results.

As is true with many problems in analysis, probabilistic approximation problems can be attacked with greater ease when they are placed within a suitably chosen abstract framework. The theory of measurable selection affords the degree of abstraction we are seeking.

In §2 we present the necessary preliminaries, including a selection theorem. In §3 we prove a selection theorem and stochastic versions of several deterministic approximation theorems.

2. Definitions, properties, and a selection theorem. If $(\Omega, \mathscr{A})$ and $(Z, \mathscr{B})$ are two measurable spaces and X is an arbitrary non-empty set, then a mapping $f\colon \Omega \times X \to Z$ is a *random function* if and only if the function $f(\cdot, x)$ is measurable for each x in X. In our applications Z is a metric space and, in this case, we choose the σ-algebra $\mathscr{B}$ as the class of Borel subsets of Z.

If X is a metric space, $(\mathscr{C}(X), H)$ is the space of compact non-empty subsets of X with the Hausdorff metric.

A relation $F\colon \Omega \to X$ is a subset of $\Omega \times X$. Alternatively, F may be regarded as a function from Ω to the set of all subsets of X. When we want to emphasize the properties of F as a subset of $\Omega \times X$, we will refer to its graph $\mathrm{Gr}(F)$ rather than F. If domain $(F) = \Omega$, then F is called a

Received by the editors on January 19, 1983.
The second author was supported in part by the National Science Foundation.