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EMBEDDINGS EXTENDING VARIOUS TYPES OF DISJOINT SETS

C.E. AULL

1. Introduction. It is well known that a set S is C^*-embedded in a set X if and only if two disjoint zero sets of S can be extended to disjoint zero of X ([7] and [4]). In the Wallman compactification [7, p. 270] two disjoint closed sets of S are extended to two disjoint closed sets of X. Here we study analogous extensions based on the sets being cozero, open, cozero and open, closed and zero and extend results on those of losed sets. The last two are related to C^*-embeddings and the first three to z-embeddings. In doing this, new characterizations of Oz-spaces, extremally disconnected spaces and modifications of these spaces will be obtained. Also, Tychonoff spaces will be characterized such that every subset (every open subset [every closed set] has a certain type of embedding property and Tychonoff spaces will be characterized such that every embedding into a Tychonoff space is of a certain type. Mappings involving these embeddings are also discussed.

In general the notation and terminology of Gillman and Jerison [7] will be used. Most of the background material on F-spaces, basically and extremally disconnected spaces, and C- and C^*-embedding will be found is this reference. Background material on z-embeddings will be found in [3] and [4]. The term normal will not necessarily include T_1.

2. Basic results.

DEFINITION 1. A space X is CC-embedded (CG-embedded) [GG-embedded] in a space Y if given two disjoint sets, both cozero (one open, one [both open] in X, they can be extended to disjoint sets, both cozero (one open, one cozero) [both open] in Y. A set B in X is extended to a set $E(B)$ in Y if $E(B) \cap X = B$.

Analogously, we define FF-embedding and FZ-embedding where F stands for a closed set and Z for a zero set.

THEOREM 1. *The following are satisfied.*

(a) *Every closed subset of a space Y is FF-embedded and every open subset of a dense set of Y is GG-embedded in the space. A set S is FF-*

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