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SURFACE FITTING WITH SCATTERED NOISY DATA ON EUCLIDEAN D-SPACE AND ON THE SPHERE

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ABSTRACT. An overview of cross validated spline methods for smoothing noisy data in the plane, in Euclidean d-space, and on the sphere is given. Cross-validated thin plate smoothing splines are reviewed and an efficient numerical algorithm for computing them for problems with up to several hundred data points is described. Some numerical results for a two-dimensional example are given. A theory of vector splines for smoothing noisy vector data on the sphere is given. The use of generalized cross-validation to estimate both the smoothing parameter as well as the relative energy to be assigned to the divergent and nondivergent part of the smoothed vector field is described and tested numerically on simulated upper air horizontal wind fields.

1. Introduction; an overview of cross-validated smoothing splines. It is assumed that data $y = (y_1, \ldots, y_n)'$ arise according to the model

$$y_i = f(P_i) + \varepsilon_i, \qquad i = 1, 2, \ldots, n,$$

where $P_i \in S$, some index set (e.g., Euclidean d-space, the sphere, torus, etc.). The function f is assumed to be a smooth function in some reproducing kernel (r.k.) Hilbert space X of real-valued or vector-valued functions on S. The $\{\varepsilon_i\}$ are independent zero mean measurement errors with common unknown variance, and it is desired to recover an estimate of f given $y = (y_1, \ldots, y_n)'$. The estimate f_λ of f will be taken as the minimizer in X of

$$(1.1) \qquad \frac{1}{n}\sum_{i=1}^{n}(y_i - f(P_i))^2 + \lambda J(f),$$

where $J(f)$ is a seminorm on X with M-dimensional null space spanned by $\phi_1, \ldots, \phi_M$, $M < n$. Here $J^{1/2}(f)$ can be taken as the norm of the orthogonal projection of f onto X_1 where X_1 is the orthocomplement of the span of the $\{\phi_\nu\}$ in X. If the $n \times M$ matrix T with (i, ν)-th entry

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