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C^1 SURFACE INTERPOLATION FOR SCATTERED DATA ON A SPHERE

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ABSTRACT. This paper describes an algorithm for constructing a smooth computable function f defined over the surface of a sphere and interpolating a set of n data values u_i associated with n locations p_i on the surface of the sphere. The interpolation function f will be continuous and have continuous first partial derivatives. The locations p_i are not required to lie on any type of regular grid.

1. Introduction. The problem of constructively defining a smooth surface that interpolates data defined at scattered points in the plane has been treated in different ways by a number of authors. For surveys of this work up to 1977 see [2] and [9].

We consider here the analogous problem for data defined at scattered points over the surface of a sphere. When data are defined over only a portion of the surface of a sphere, it may be most reasonable to map that portion of the spherical surface to a planar region, using a C^1 mapping function, and treat the problem by an algorithm designed for the planar domain problem. However, when the data are scattered over the whole surface, and it is desired to produce a C^1 interpolation function defined over the entire surface, it seems necessary, or at least very desirable, to deal with the problem directly in the spherical setting. In particular, there is no C^1 function that will map the entire surface of a sphere to a bounded planar region.

2. The problem. Let S denote the surface of the unit sphere in three-space. Given points p_i, $i = 1, \ldots, n$, the problem is to construct a computable function f defined and having C^1 continuity over S and satisfying the interpolation conditions $f(p_i) = u_i$ for $i = 1, \ldots, n$.

2.1. Relevant properties of C^1 functions on S. A function, f, defined on S is differentiable at a point p_0 in S if and only if there exists a three-vector g_0 satisfying

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