ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 1, Winter 1984

A TRANSFINITE C^2 INTERPOLANT OVER TRIANGLES

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ABSTRACT. A transfinite C^2 interpolant on a general triangle is created. The required data are essentially C^2, no compatibility conditions arise, and the precision set includes all polynomials of degree less than or equal to eight. The symbol manipulation language REDUCE is used to derive the scheme. The scheme is discretized to two different finite dimensional C^2 interpolants in an appendix.

1. Introduction and history. Scientists and engineers often take three-dimensional measurements through which they wish to pass a surface. When designing interactively the surface of a real object, designers input three-dimensional points. Because the geometric information for these two classes of problems can be located arbitrarily in three-dimensional space, the surface scheme must be able to handle arbitrarily located data. There are two broad classes of methods suitable for solving these problems (i.e., problems in which simplifying geometric assumptions cannot be made): (1) patch methods, and (2) point methods. "Patch methods" are those methods in which small curved pieces are joined together to form a smooth surface. "Point methods" are those methods in which information given only at discrete points is used to construct a surface.

This paper and its appendix introduce new patch methods which have the following properties: (1) the data may be arbitrarily located, and (2) the interpolating surface is twice continuously differentiable (C^2). We have divided the development of our new schemes into two parts. This paper is Part 1 and the accompanying appendix by Alfeld is Part 2.

(a) In Part 1, we develop schemes of interpolation to *curves* of information defined over triangles. (These are called *transfinite* interpolants because entire curves of information are interpolated.)

(b) In Part 2, we discretize these transfinite interpolants to obtain finite dimensional patches (i.e., patches which depend on only finitely many data). A reason for developing transfinite patches per se is that there is a unified theory of the interpolation properties, polynomial precision, and

This research was supported in part by The National Science Foundation with Grant MCS-8101854 and by The Department of Energy with Contract DE-AC02-82ER12046. A000 to The University of Utah.