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A REPRESENTATION FOR THE CONSTRUCTIVE DESIGN OF CURVED SURFACES

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ABSTRACT. Traditionally, there have been two major methods for the representation of complex, curvilinear shapes. Objects are represented by binary trees of set operations upon a restricted set of rigid forms, such as cylinders, spheres, etc., or by an explicit piecewise definition of the boundary by polygons and patches. The former is intuitive for modeling, but lacks generality, particularly in the area of smoothness. The latter, though sufficiently general for representation, is often difficult to model in environments where properties such as symmetry are as important as interpolatory constraints. This paper presents a functional representation, called Constructive Surface Geometry, which unifies both approaches and which possesses advantages of each.

Introduction. Currently, most solid, animation and simulation production modeling systems are not based upon a primitive capable of representing smooth, curved surfaces. [5] There are several reasons for this.

1. It is much more difficult to render smooth shapes than faceted ones. The convex planar polygon has a unique property; its perspective projection is always silhouetted by the projection of its boundary. This is not true of any curved surface, where determination of the silhouette after perspective projection entails the solution of a non-linear equation.

2. The increase in computation required to manipulate curved objects is very large. Calculation of intersections of faceted objects leads to linear systems; calculation of intersections of curved objects requires solution of non-linear systems.

3. While it is well known that any continuous surface can be uniformly approximated to within arbitrary $\varepsilon > 0$ by a network of patches, this says nothing about the ability of a designer to model curved features in a particular system. [3] In fact, designing by interpolatory constraints alone, even in an interactive graphical environment where dynamic rendering of three-dimensional objects is possible, is often difficult. This follows from the necessity of modeling through a two-dimensional interface and either using several orthographic projections, or a single perspective projection. The limitations of human depth perception are all too