ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 1, Winter 1984

THREE-STAGE INTERPOLATION TO SCATTERED DATA

THOMAS A. FOLEY

1. Introduction. A general three-stage procedure is presented here that solves the following multivariate interpolation problem: Let D be a subset of $\mathbf{R}^n$ that contains N distinct point v_i. Given N real values $f(v_i)$, construct a function $P[f]$ defined on D that satisfies $P[f]\ (v_i) = f(v_i)$ for $i = 1, \ldots, N$.

In the bivariate case with $n = 2$, this problem can be interpreted as fitting a surface through N points in three-dimensional space. Foley [7] used bivariate interpolation in the characterization of radionuclide activity resulting from nuclear tests in Nevada. The survey paper by Schumaker [14] gives applications in mineral exploration, medicine, computer aided design, and electronics.

Part of the motivation for three-stage interpolation is that some methods that apply directly to scattered data give undesirable results or they are inefficient when N is large. On the other hand, many methods that are accurate and efficient only apply to gridded data, which is a narrower class of data.

Three-stage interpolation is related to the following approaches. Schumaker [13] gives a two-stage approximation to the scattered data that does not generally solve the interpolation problem. The Boolean sum approach in Barnhill and Gregory [3] obtains a desired precision and retains the interpolation properties. The implementation of the Barnhill-Gregory Boolean sums by Poeppelmeier [12] is discussed in Barnhill [2]. The author in [6] and Foley and Nielson [8] used delta sums and delta iteration interpolants composed of a bicubic spline approximation and a correction term using Shepard's method. It will be shown that some of these approaches also can be classified as three-stage methods.

The next section describes general three-stage interpolation. §3 gives a bivariate example named BSPLASH that is globally defined and has continuous second order partial derivatives. This method is applied to several data sets in the final section, and the results compare favorably with the best methods tested in Franke [9].

2. General three-stage interpolation. The general three-stage process is defined by