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A STORAGE-EFFICIENT METHOD FOR CONSTRUCTION OF A THIESSEN TRIANGULATION

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ABSTRACT. This paper describes a storage-efficient method and associated algorithms for constructing and representing a triangulation of arbitrarily distributed points in the plane.

1. Introduction. This paper addresses the following problem. Given a set of nodes (X_i, Y_i), $i = 1, \ldots, N$, arbitrarily distributed in the $X - Y$ plane, construct a triangulation with the nodes as vertices and which is as nearly equiangular as possible. The primary application of such a triangulation is as a preliminary step in a triangle-based method for bivariate interpolation of data values associated with the nodes [7], [5], [1], [2]. The triangulation also serves as an efficient mechanism for solving closest-point problems such as finding the two closest nodes and finding a largest circle containing none of the nodes. These problems arise in a variety of applications, e.g., wire layout, clustering, facilities location, and constructing the feasible polygon for linear programming in two variables with N constraints [13]. Another application of the triangulation method is as an automatic mesh generator for a triangle-based finite element code. Lists of element node numbers and boundary node numbers can be generated from a set of nodal coordinates which are concentrated in regions where the solution varies most rapidly. In addition to generating the mesh, the method guarantees its validity. Simpson [16] provides and discusses the importance of an algorithm which verifies the consistency of a set of finite element input data.

This paper describes the algorithms implemented in an extensive and well-documented software package for triangulation and interpolation written in a subset of ANSI standard FORTRAN accepted by the PFORT verifier [12]. The software listing is found in Renka [9], and machine-readable code may be obtained from the second author. The primary goal of this research was a triangulation package requiring less storage than those previously available. This was achieved at the cost of a relatively small loss in time efficiency over alternative packages.

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