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FINITE HARMONIC AND GEOMETRIC INTERPOLATION

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1. Introduction. In the works [4] and [5], the authors have been developing the theory of finite harmonic interpolation in the unit disk. The basic idea is to express the value of a real-valued harmonic function u in the disk as a finite weighted mean

$$u(z) = \frac{1}{N} \sum_{k=1}^{N} \frac{R^2 - |z|^2}{|\zeta_k - z|^2} u(\zeta_k), \tag{1}$$

for $|z| < R < 1$, $\zeta_1, \zeta_2, \ldots, \zeta_N$ points equally spaced on $|z| = R$, and N a fixed positive integer.

In the present work, we also consider the notion of finite harmonic interpolation on a general domain Ω with an exhaustion $\{\Omega_n\}$ such that the boundary of each Ω_n, $\partial\Omega_n$, is an analytic Jordan curve. The Green's function $g_n(z, \zeta)$ of Ω_n with pole z has an inner normal derivative $\partial g/\partial\eta$ and each Ω_n has length L_n.

If u is a real-valued harmonic function on Ω and z is in Ω_n then

$$u(z) = \frac{1}{2\pi} \int_{\partial\Omega_n} u(\zeta) \frac{\partial g_n(z, \zeta)}{\partial\eta} |d\zeta|, \tag{2}$$

and $\partial g_n(z, \zeta)/\partial\eta$ is continuous on the analytic Jordan curve $\partial\Omega_n$. Since $\int_{\partial\Omega_n} u(z)\, |d\zeta| = L_n u(z)$ we can rewrite equation (2) to obtain

$$\int_{\partial\Omega_n} \left[u(\zeta) \frac{L_n}{2\pi} \frac{\partial g_n(z, \zeta)}{\partial\eta} - u(z)) \right] |d\zeta| = 0. \tag{3}$$

Let $F(\zeta) = u(\zeta)\,(L_n/2\pi)(\partial g_n(z, \zeta)/\partial\eta - u(z))$ and parametrize ζ in terms of arc length s, say $\zeta = \psi(s)$. Also let $\partial\Omega_n = \bigcup_{k=1}^{N} \gamma_k$, where each segment γ_k has length L_n/N, and denote by $F_k(s)$, $0 \leqq s \leqq L_n/N$, the restriction of $F(\psi(s))$ to γ_k. Then from (3),

$$\int_0^{L_n/N} \left[\sum_{k=1}^{N} F_k(s) \right] ds = 0.$$

By the continuity of F there exists s_0 such that $\sum_{k=1}^{N} F_k(s_0) = 0$. That is, there exist N equally spaced points $\zeta_1, \zeta_2, \ldots, \zeta_N$ on $\partial\Omega_n$ such that

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