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Q-ANALOGUE OF A TRANSFORMATION OF WHIPPLE

A. VERMA AND V.K. JAIN

1. Whipple [**18**] obtained two transformations between nearly-poised hypergeometric series (second kind) ${}_4F_3$ and Saalschützian hypergeometric series ${}_5F_4$.

$$(1.1)\quad {}_4F_3\left[\begin{matrix} a, b, c, -m; \\ 1+a-b, 1+a-c, w\end{matrix}\right]$$

$$= \frac{(w-a)_m}{(w)_m}\,{}_5F_4\left[\begin{matrix} 1+a-w, \frac{a}{2}, \frac{1+a}{2}, 1+a-b-c, -m; \\ 1+a-b, 1+a-c, \frac{1}{2}(1+a-w-m), \frac{1}{2}(2+a-w-m)\end{matrix}\right]$$

and

$$(1.2)\quad {}_4F_3\left[\begin{matrix} -n, b, c, d; \\ 1-n-b, 1-n-c, w\end{matrix}\right]$$

$$= \frac{(w-d)_n}{(w)_n}\,{}_5F_4\left[\begin{matrix} d, 1-n-b-c, \frac{-n}{2}, \frac{1-n}{2}, 1-n-w; \\ 1-n-b, 1-n-c, \frac{1}{2}(1+d-w-n), \frac{1}{2}(2+d-w-n)\end{matrix}\right].$$

It is easy to note that (1.2) follows from (1.1) (and vice-versa). Indeed in (1.1) setting $a = -n$ and $m = -d$ we get (1.2) with the restriction $d = -m$. The condition on d can be waived by analytic continuation because both sides are polynomials in d.

In 1929 Bailey [**6**] obtained a transformation between a nearly-poised series ${}_5F_4$ and Saalschützian ${}_5F_4$ which is analogous to (1.1), viz.,

$$(1.3)\quad {}_5F_4\left[\begin{matrix} a, 1+\frac{a}{2}, b, c, -m \\ \frac{a}{2}, 1+a-b, 1+a-c, w\end{matrix}\right] = \frac{(w-a-1-m)\,(w-a)_{m-1}}{(w)_m}$$

$$\cdot\,{}_5F_4\left[\begin{matrix} 1+\frac{a}{2}, \frac{1+a}{2}, 1+a-b-c, 1+a-w, -m; \\ 1+a-b, 1+a-c, \frac{1}{2}(2+a-w-m), \frac{1}{2}(3+a-w-m)\end{matrix}\right].$$