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DUALITY FOR INFINITE HERMITE SPLINE INTERPOLATION

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1. Introduction. Let $x = (x_i)_{-\infty}^{\infty}$, $\xi = (\xi_i)_{-\infty}^{\infty}$ be non-decreasing sequences in $\mathbf{R}$ satisfying

$$(1) \qquad |\{i \mid x_i = t\}| + |\{i \mid \xi_i = t\}| \leqq n + 1,$$

where $|S|$ denotes the number of elements in a set S.

For a positive integer n, we denote by (n, x, ξ) the problem of interpolatting data at x by spline functions of degree n with knots at ξ. To make this precise we define for each integer i,

$$(2) \qquad \mu_i = |\{k < i \mid x_k = x_i\}|, \nu_i = |\{k < i \mid \xi_k = \xi_i\}|.$$

Then the space of spline functions of degree n with knots at ξ is defined to be

$$\begin{aligned} \zeta_n(\xi) := \{f: (\xi_{-\infty}, \xi_\infty) \to \mathbf{R} \mid & \text{ for any integer } i \text{ with} \\ & \xi_i < \xi_{i+1}, f \text{ coincides on } (\xi_i, \xi_{i+1}) \text{ with a} \\ & \text{polynomial of degree } \leqq n \text{ and } f^{(j)} \text{ is continuous} \\ & \text{at } \xi_i, 0 \leqq j \leqq n - \nu_i - 1\}, \end{aligned}$$

where $\xi_{\pm\infty} = \lim_{i\to\pm\infty} \xi_i$.

We shall say (n, x, ξ) is *solvable* if for any bounded sequence $(y_i)_{-\infty}^{\infty}$ in $\mathbf{R}$ there is a unique bounded spline f in $\zeta_n(\xi)$ satisfying

$$(3) \qquad f^{(\mu_i)}(x_i) = y_i \ (i \in \mathbf{Z}).$$

For this to make sense we must have $x_i \in (\xi_{-\infty}, \xi_\infty)$ $(i \in \mathbf{Z})$.

We note that condition (1) ensures that we do not interpolate at a discontinuity. Defining

$$(4) \qquad \Delta x_i = \min\{x_j - x_i \mid x_j > x_i\},$$

we define the *global mesh ratio* of x as

$$(5) \qquad \sup\{\Delta x_i / \Delta x_j \mid i, j \in \mathbf{Z}\}.$$

A similar definition holds for ξ. We shall prove the following.

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