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ON ROTA'S MODELS FOR LINEAR OPERATORS

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ABSTRACT. In this note Rota type models for linear operators in terms of weighted shifts are obtained.

The role of shift operators in the structure theory of Hilbert space operators is well known. It was first pointed out by Rota [6] that they can be regarded as 'universal' operators, see also de Branges and Rovnyak [1] and Foias [2]. The object of this note is to obtain Rota type theorems for weighted shift operators and to generalize some of the known results in this direction. For a beautiful and almost exhaustive account of the literature on weighted shifts through 1973, we refer to Shields [7]. However, the aspect of their study which forms the subject of this note has not been touched there. A model for quasinilpotent operators in terms of weighted shifts has been obtained by Foias, and Pearcy [3].

Let H be an infinite-dimensional, separable complex Hilbert space with an orthonormal basis $\{e_n\}_{n=0}^{\infty}$. We shall denote by $B(H)$ the algebra of all bounded operators on H. Let $\alpha = \{\alpha_n\}_{n=1}^{\infty}$ be a bounded sequence of complex numbers. The operator S_α on H defined by

$$S_\alpha e_0 = 0 \text{ and } S_\alpha e_n = \alpha_n e_{n-1} \text{ for } n \geq 1$$

is called the backward weighted shift with the weight sequence α. If $\alpha_n = 1$ for all n, then S_α is simply called the backward shift which we shall denote by S. We may and shall take α to be a bounded sequence of positive real numbers [4], [5]. If $\ell^2(H)$ denotes the Hilbert space of all square-summable sequences $x = \{x_0, x_1, \ldots, x_n, \ldots\}$ of vectors x_n's in H, then S_α on $\ell^2(H)$ appears as

$$S_\alpha(x) = \{\alpha_1 x_1, \alpha_2 x_2, \ldots, \alpha_{n+1} x_{n+1}, \ldots\}.$$

We write $\beta_n = \alpha_1 \alpha_2 \ldots \alpha_n$ for $n \geq 1$ and $\beta_0 = 1$. We shall denote by $r(T)$ the spectral radius of an operator T in $B(H)$. A subspace M of H is

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