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LOCALLY Σ-SPACES

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Introduction. Σ-spaces and reduced Σ-spaces were introduced by 0. Loos [**11**]; they include reflexion spaces [**7**, **8**, **9**] and symmetric spaces where the group Σ is just $\mathbf{Z}_2$, also s-manifolds [**1**, **5**, **6**] where Σ is cyclic. The main purpose of this paper is to define local analogues of (reduced) Σ-spaces and to show that certain desirable properties are then satisfied. Thus, we would expect that, in addition to the above, such spaces should include locally symmetric spaces. Also, with suitable restrictions on Σ, there should be an extension to the theorem that a locally symmetric space admits a connection invariant by local symmetries [**10**].

Theorem 1 in §3 contains the essential results with respect to the above remarks, where Σ is assumed to be cyclic or compact. It also indicates the importance of those cases where an affine connection related to the local structure exists. Such spaces are introduced in §4, and Theorem 2 provides an alternative characterisation of them. This leads, in Theorem 3, to a further simplification of affine locally Σ-spaces under assumptions of analyticity. In particular, as shown in the corollary to Theorem 3, such analytic conditions always hold when Σ is cyclic or compact, Finally, we obtain alternative forms of Theorem 3 in which only tensor properties are required. Further results on these local structures will appear in a later paper.

1. Notation. Manifolds are always finite dimensional and smooth unless otherwise stated. For any smooth map ϕ between manifolds we again write ϕ (or perhaps $T\phi$) for its differential. For any manifold M, $\mathscr{X}(M)$ is the Lie algebra of smooth vector fields on M, TM (resp. T_xM) is the tangent bundle over M (resp. the tangent space to M at x), and $T_1^1(M)$ is the (1, 1) tensor bundle over M. The tangent bundle over a product $M \times N$ is identified with $TM \times TN$ in the usual way.

For manifolds M, N we write $\phi\colon M \rightarrowtail N$ for any smooth map ϕ with domain some subset U of M, and write $\boldsymbol{\phi}_x$ for the germ of ϕ at x. Let $G(M, N)$ denote the set of all germs $\boldsymbol{\phi}_x$, $x \in M$. The projection π: $G(M, N) \to M$ is defined by $\boldsymbol{\phi}_x \to x$, and the evaluation map ν: $G(M, N) \to N$ by $\nu(\boldsymbol{\phi}_x) = \phi(x)$. To each ϕ there corresponds a section $\boldsymbol{\phi}$; $x \to \boldsymbol{\phi}_x$, and a topology is defined on $G(M, N)$ by the condition that sets im $\boldsymbol{\phi}$ should

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