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ON THEOREMS OF B.H. NEUMANN CONCERNING FC- GROUPS, II

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ABSTRACT. B.H. Neumann characterized central-by-finite and finite-by-abelian groups. In this paper, we generalize these results by characterizing those members of a large class of groups that are central-by $(<m)$ or $(<m)$-by-abelian. Here we mean that a group is $(<m)$ if its cardinality is less than m for some infinite cardinal m.

1. Introduction. B.H. Neumann [8] characterized central-by-finite and finite-by-abelian groups. A group G is central-by-finite if and only if each subgroup has only finitely many conjugates or, equivalently, U/U_G is finite for each subgroup U of G. Here U_G denotes the core of U; that is, the largest normal subgroup of G contained in U. We use U^G to denote the normal closure of U in G; then G is finite-by-abelian if and only if $|U^G: U|$ is finite for each subgroup U of G.

Eremin [3] indicated that it is only necessary to consider the abelian subgroups of G in the first of these results. A corrected form of Eremin's proof can be found in the book by Gorčakov [7].

In [13], one of us considered the extension of these results to FC-groups in which $|G/Z(G)| < m$ or $|G'| < m$, where m denotes an infinite cardinal. Here we go further and consider the extent to which the FC-condition can be relaxed.

To describe our results, we define the following classes of groups. If m is an infinite cardinal, the class mC consists of those groups G in which $|G: C_G(x)| < m$ for each $x \in G$. Z_m is the subclass of mC consisting of those groups G in which $|G: C_G(S)| < m$ for each subset $S \subseteq G$ such that $|S| < m$. See [4] and [5] for theorems concerning the abelian subgroup structure of mC-groups.

In the case $m = \aleph_0$, both these classes coincide with the class of FC-groups and so either class may be considered as a generalization of the class of FC-groups. As was shown in [13], the condition on Z_m-groups makcs these groups much easier to work with, and here we are able to prove the following results.

*The major portion of this work was done while the first author was on the faculty of the University of Colorado at Denver.

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