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KRULL DIMENSION OF DIFFERENTIAL OPERATOR RINGS II: THE INFINITE CASE

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In a recent paper [**2**], Goodearl and Warfield have considered the problem of computing the Krull dimension of the differential operator ring $R[\theta;\delta]$, when R is a commutative Noetherian ring with a derivation δ. They have given a reasonably complete description in the case that K.dim(R) is finite, but have only obtained partial results in the infinite case. Here we obtain a description of the infinite case that parallels the results of Goodearl and Warfield in the finite case. The notations and definitions of [**2**] will be used here and the reader is recommended to have a copy of that paper at hand since the proofs in this paper rely heavily on the methods of [**2**].

Throughout the paper, R will be a commutative Noetherian ring and δ a derivation on R. The differential operator ring $R[\theta;\delta]$ will be denoted by T.

The major result of [**2**] shows that if R has finite Krull dimension n then K.dim$(T) = n$ except when there is a maximal ideal M of height n with $\delta(M) \subseteqq M$ or char$(R/M) > 0$, in which case K.dim$(T) = n + 1$. Example 4.7 of [**2**] shows that the maximal ideals of R do not control the Krull dimension of T in the case that K.dim(R) is infinite. It will be shown here that if K.dim$(R) = \eta + n$, where η is a limit ordinal and n a natural number, then it is the prime ideals M such that K.dim$(R/M) = \eta$ that control the Krull dimension of T. For this reason we begin with a careful analysis of the limit ordinal case.

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THEOREM 1. *Let x be a non-zero divisor in R. Let R_x and R_C denote the localisations at the denominator sets $\{x^n \mid n = 0, 1, 2, \ldots\}$ and $C = \{1 - xr \mid r \in R\}$ of R.*

(i) *$\{x^n\}$ and C are denominator sets in T, and δ extends to the localisations of R by the quotient rule; so that there are natural isomorphisms $T_x \cong R_x[\theta;\delta]$ and $T_C \cong R_C[\theta;\delta]$.*

(ii) *The diagonal map from T to $T_x \oplus T_C$ is a faithfully flat embedding.*

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