ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 13, Number 3, Summer 1983

A q-EXTENSION OF BAILEY'S BILINEAR GENERATING FUNCTION FOR THE JACOBI POLYNOMIALS

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ABSTRACT. This note presents a rather simple proof of an interesting q-extension of Bailey's bilinear generating function for the classical Jacobi polynomials. The proof given here uses only such elementary results as the q-analogues of Euler's transformation, Vandermonde's summation theorem, and binomial expansion.

1. Introduction. Several interesting proofs are given in the literature for Bailey's bilinear generating function for the classical Jacobi polynomials [**1**, p. 9, Eq. (2.1)]. One of the recent proofs is given by Stanton [**3**]; it uses the orthogonality property of Jacobi polynomials and a known quadratic transformation for a well-poised hypergeometric ${}_3F_2$ series. Indeed, as remarked by Stanton [**3**, p. 399], this technique applies *mutatis mutandis* to yield a q-extension of Bailey's result. The object of the present note is to give a rather simple proof of the q-extension, using only such elementary results as the q-analogues of Euler's transformation, Vandermonde's summation theorem, and binomial expansion.

2. Definitions and preliminaries. Put

$$(2.1)\qquad (\lambda; q)_n = \begin{cases} 1, & \text{if } n = 0, \\ (1-\lambda)(1-\lambda q)\dots(1-\lambda q^{n-1}), & \forall n \in \{1, 2, 3, \dots\}, \end{cases}$$

and let ${}_{p+1}\Phi_p$ denote the standard q-hypergeometric series with $p + 1$ numerator and p denominator parameters. Then, in terms of the little q-Jacobi polynomials defined by

$$(2.2)\qquad p_n^{(\alpha,\beta)}(x; q) = \frac{(\alpha q; q)_n}{(q; q)_n}\, {}_2\Phi_1\left[\begin{matrix} q^{-n}, \ \alpha\beta q^{n+1}; \\ \alpha q; \end{matrix}\ q, qx\right],$$

Supported, in part, by NSERC (Canada) grant A-7353.
1980 *Mathematics subject classification*: Primary 33A30, 33A65; Secondary 42C10.
Key words and phrases: Bilinear generating function, Jacobi polynomials, orthogonality property, quadratic transformation, well-poised hypergeometric series, Euler's transformation, Vandermonde's summation theorem, binomial expansion, q-hypergeometric series, q-Jacobi polynomials, Heine's theorem.
Received by the editors on August 31, 1981, and in revised form on September 20, 1982.