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ON THE STRUCTURE OF TORCH RINGS

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Torch rings appeared for the first time in the literature, under the name of ?-rings, when it was tried to characterize the commutative rings whose finitely generated modules are direct sums of cyclic modules ([**8**]; see [**4**] for an exposition of the history of the problem and the techniques with which it has been solved). Later torch rings have also appeared in [**10**] in the study of the commutative rings R with the property that the total ring of fractions of R/I is self-injective for all ideals I of R. In this paper we study the structure of torch rings and give an example of a torch ring which is not a trivial extension. This answers a question posed by T. Shores and R. Wiegand in [**8**].

A commutative ring R with identity is a *torch ring* if 1) R is not local, 2) the nilradical $N(R)$ of R is a prime ideal and is a non-zero uniserial R-module, 3) $R/N(R)$ is an h-local domain, and 4) R is a locally almost maximal Bézout domain (see [**4**] for the terminology.) Shores and Wiegand constructed a torch ring which was a *trivial extension*. Recall that an *extension* of the ring S by the S-module N is an exact sequence of abelian groups

$$0 \longrightarrow N \xrightarrow{i} R \xrightarrow{p} S \longrightarrow 0,$$

where R is a ring and p is a ring homomorphism such that $r \cdot i(x) = i(p(r) \cdot x)$ for all $r \in R$, $x \in N$. An extension

$$0 \longrightarrow N \xrightarrow{i} R \xrightarrow{p} S \longrightarrow 0$$

of the ring S by the S-module N is *trivial* if there exists a ring homomorphism $g: S \to R$ with $p \circ g = 1_S$ [**1**, Ch. 16].

Shores and Wiegand [**8**] have asked whether every torch ring R was a trivial extension of the ring $R/N(R)$ by its nilradical $N(R)$. In the first part of this paper we construct a torch ring R of characteristic p^2, where p is a prime; if R has characteristic p^2, the domain $R/N(R)$ must have characteristic p, so that there do not exist homomorphisms $R/N(R) \to R$.

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