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MARKOV OPERATORS AND INVARIANT BAIRE FUNCTIONS

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ABSTRACT. Let X be a compact T_2-space and T a Markov operator on $C(X)$, i.e., $T \geqq 0$ and $T1 = 1$. We deal with Baire functions which are almost everywhere T-invariant, and focus on conditions under which such Baire functions belong to the smallest system of Baire functions containing the continuous invariant functions, and closed under almost everywhere convergence. Our main result is that this occurs precisely when both T and T^* are strongly ergodic. We also show that almost everywhere invariant Baire functions separate the invariant probabilities, although everywhere invariant Baire functions need not.

1. Introduction. We define $F(T) = \{f \text{ in } C(X): Tf = f\}$, $F(T^*) = \{m \text{ in } C(X)^*: T^*m = m\}$, and $P(T^*) = F(T^*) \cap$ probabilities. T is called *strongly ergodic* if for each f in $C(X)$, $T_n f$ converges in the Banach space $C(X)$, where $T_n = (1/n)(I + \cdots + T^{n-1})$. Recall that T is s.e. if and only if $F(T)$ separates $F(T^*)$ if and only if $F(T)$ separates $P(T^*)$ [7, Theorems 2.2 and 2.7].

Let B be the set of Baire functions, so $B = \bigcup\{B_a: a < \omega_1\}$, where ω_1 is the first uncountable ordinal and B_a the a-th Baire class. We define $B(F(T))$ to be the smallest set of bounded functions containing $F(T)$, and closed under bounded pointwise sequential convergence. If f and g are in B, we say $f = g$ $P(T^*)$-ae if $f = g$ m-ae for all m in $P(T^*)$. Let $B(F(T))^a$ be the smallest set of bounded Baire functions containing $F(T)$, and closed under $P(T^*)$-ae convergence.

It will be convenient to extend the operator T to an operator (again called T) on the Baire functions by letting $Tf(x) = \int f d(T^*\delta_x)$, where δ_x is the Dirac measure at x. An easy transfinite induction over the Baire classes shows that if f is in B, then so is Tf. The same argument shows that if f is in B and m in $F(T^*)$, then $\int f dm = \int Tf dm$.

We shall need the following known result.

THEOREM. [1, Proposition 2.1]. *The following are equivalent.*

(i) *Both T and T^* are strongly ergodic.*

(ii) $F(T^{**}) = \sigma(C(X)^{**}, C(X)^*)$*-closure of* $F(T)$.

(iii) *Norm-closure*$(I - T)^*(C(X)^*) =$ *weak-*closure*$(I - T)^*(C(X)^*)$.

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