ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 13, Number 3, Summer 1983

LIMIT BEHAVIOR OF SOBOLEV TOTAL FLUX BOUNDARY CONTROL PROBLEMS

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ABSTRACT. We consider control of certain nonlocal boundary value problems of Sobolev equations. Regularity results are obtained for the optimal controls. Further, convergence results are obtained for the solution of the boundary value problems as well as the control problem as the equations formally approach the diffusion equation.

1. Introduction. Let D be a nonempty bounded domain in $\mathbf{R}^n$, $n = 2$ or 3, with smooth boundary Γ. Let $a \in D$ and $B(a, \rho)$ be the ball centered at a with radius $\rho > 0$ and boundary Γ_ρ such that $B(a, \rho) \subset D$. Let $\Omega = D - B(a, \rho)$ so that $\partial\Omega = \Gamma_\rho \cup \Gamma$. Finally, let $Q = \Omega \times (0, T)$ with $\Sigma = \Gamma \times (0, T)$ and $\Sigma_\rho = \Gamma_\rho \times (0, T)$. We study control problems governed by the following nonlocal boundary value problem

$$(1) \qquad (1 - \varepsilon\Delta)y_t^{(\varepsilon)} - \Delta y^{(\varepsilon)} = 0 \text{ in } Q$$

$$(2) \qquad y^{(\varepsilon)}(x, 0) = 0 \text{ in } \Omega$$

$$(3) \qquad y^{(\varepsilon)}(x, t) = 0 \text{ on } \Sigma$$

$$(4) \qquad y^{(\varepsilon)}(x, t) = C_\varepsilon(t) \text{ on } \Sigma_\rho$$

where $C_\varepsilon(t)$ is an unknown function of t only

$$(5) \qquad \int_{\Gamma_\rho} \frac{\partial}{\partial n}(y^{(\varepsilon)} + \varepsilon y_t^{(\varepsilon)})d\sigma = v(t) \text{ a.e. in } [0, T]$$

where $v \in L^2(0, T)$.

In equation (1) the Laplacian may be replaced by another operator A that is second order symmetric uniformly strongly elliptic in Ω. Equation (1) with $\varepsilon > 0$ arises in the modelling of many physical phenomena [**2**]. Of particular interest is the flow of a fluid through fissured media [**1**]. In this case a system of fractures is assumed that separates blocks of a porous media thereby creating a material having two porosities. The solution y of equation (1) at a point represents an average pressure of the

This work was supported in part by a National Science Foundation Grant No. MCS-7902037.

Received by the editors on May 10, 1982.