ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 13, Number 2, Spring 1983

ASYMPTOTIC BEHAVIOUR OF A REACTION-DIFFUSION EQUATION IN HIGHER SPACE DIMENSIONS

CHRISTOPHER K.R.T. JONES

ABSTRACT. The reaction-diffusion equation considered has a travelling wave solution in one space dimension for which strong stability results have been proved by Fife and McLeod [3]. In this paper it is proved that a certain class of solutions of this equation, in higher space dimensions, approach this one-dimensional travelling wave when followed out along any ray.

1. Introduction. In this paper I extend a theorem of Jones [4]. The result concerns the reaction-diffusion equation:

$$u_t = \Delta u + f(u), \tag{1.1}$$

where $u \in \mathbf{R}$, $x = (x_1, \ldots, x_n) \in \mathbf{R}^n$ and $\Delta = \partial^2/\partial x_1^2 + \cdots + \partial^2/\partial x_n^2$. Here f: $\mathbf{R} \to \mathbf{R}$ is assumed to be smooth and to have the cubic-like form depicted in Fig. 1.

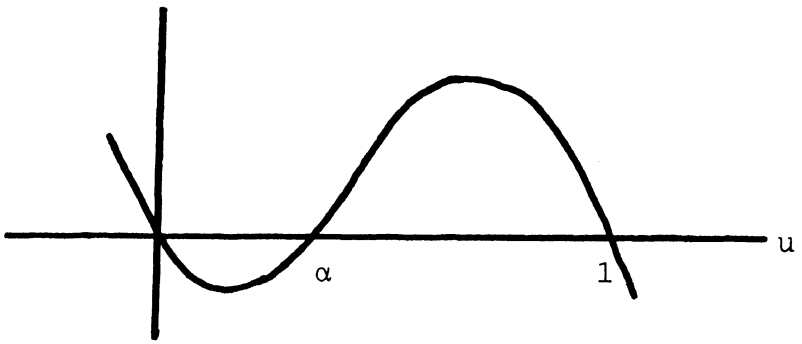

Fig. 1

Specifically it has three zeroes 0, α and 1, with $f'(0) < 0$, $f'(1) < 0$ and $\int_0^1 f(u)du > 0$. An initial value problem is naturally associated with (1.1):

The research is partially supported by an NSERC operating grant No. 67-9215.
Received by the editors on February 1, 1982.