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ON THE GROTHENDIECK AND NIKODYM PROPERTIES FOR ALGEBRAS OF BAIRE, BOREL AND UNIVERSALLY MEASURABLE SETS

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ABSTRACT. Let $\mathscr{A}$ be a Boolean algebra, represented as the algebra of clopen subsets of a zero-dimensional compact Hausdorff space T. Let $C(T)$ be the Banach space of continuous scalar-valued functions on T, $M(T)$ its dual space. Then $\mathscr{A}$ has the Grothendieck property (G) if every weak* convergent sequence in $M(T)$ is weakly convergent; $\mathscr{A}$ has the Nikodym property (N) if every subset of $M(T)$ which is setwise bounded on $\mathscr{A}$ is uniformly bounded.

Recently Schachermayer has shown that the algebra of Jordan measurable subsets of $[0, 1]$ has (N), but fails (G). In this paper the universal measure space of Graves is used to place Schachermayer's result in a more general context. Various "Jordan algebras" of subsets of a given T, for example the algebra J of Baire sets with scattered boundary, are examined with respect to properties (G) and (N). The results include: (a) If T is an F-space, then J has both (G) and (N); (b) If T is first countable, then J has (N); (c) If T is metrizable and not scattered, then J fails (G); and (d) If $T = 2^A$, A uncountable, then J fails both (G) and (N). The Alexandroff duplicate of a given T, and the notion of a quasi-F-space introduced by Dashiell play a prominent role in the discussion. Some applications to vector measures with range in a Fréchet space are also given.

1. Introduction. This paper studies Boolean algebras for which analogues of the classical Nikodym and Vitali-Hahn-Saks Theorems hold for sequences of bounded additive measures. The Grothendieck property (that every weak*-convergent sequence in the dual of a Banach space should be weakly convergent) is also investigated in this context. The work builds on recent, fundamental progress in the area due to Schachermayer **[29]**. The approach is via the universal measure space of Graves **[14]**, using continuity and orthogonality properties of vector measures studied by Brook **[2, 3]**. The results for scalar measures then follow from topological properties of the Stone space of a given Boolean algebra.

A history of the Grothendieck, Nikodym, and Vitali-Hahn-Saks properties can be found in the book by Diestel and Uhl **[9]**. See **[5, 7, 8, 10, 11, 35]** for recent work in this area. A starting point for this paper is Schachermayer's discovery **[29]** that the algebra of Jordan measurable sub-

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