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ON DIOPHANTINE EQUATIONS OF THE FORM $1 + 2^a = p^b q^c + 2^d p^e q^f$

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ABSTRACT. In this paper several Diophantine equations of the form $1 + 2^a = p^b q^c + 2^d p^e q^f$, where p and q are distinct odd primes, and the exponents are non-negative integers are solved. In particular this equation is solved for $(p, q) = (23, 47)$, $(7, 23)$, and $(73,223)$. The related equations $1 + 73^a = 2^b 223^c + 2^d 73^e 223^f$ and $1 + 223^a = 2^b 73^c + 2^d 73^e 223^f$ are also solved. This work extends recent work of the authors and J.L. Brenner.

1. Introduction. In this paper we consider equations of the form

$$1 + x = y + z, \tag{1}$$

where the primes dividing xyz are specified. Such equations are exponential Diophantine equations. For example if the primes dividing xyz in (1) are 2, 3, and 5 and $(x, 15) = 1$, then (1) has the form

$$1 + 2^a = 3^b 5^c + 2^d 3^e 5^f, \tag{2}$$

where a, b, c, d, e and f are non-negative integers. Thus it is the exponents a, b, c, d, e and f which are to be determined.

These equations (1) and (2) are special cases of the general equation $\sum x_i = 0$, $i = 1, 2, 3, \ldots, m$, where the primes dividing $x_1 x_2 \cdots x_m$ are specified. There has been very little work done in general to solve such equations. For example the equation

$$1 + 3^a = 5^b + 3^c 5^d \tag{3}$$

is unsolved. Also it is unknown whether such equations always have a finite number of non-trivial solutions. Such equations always have an infinite number of trivial solutions. For example the equation (3) above has infinitely many solutions of the form $b = d = 0$ and $a = c$.

It follows from work of Dubois and Rhin [5] and Schlickewei [6] that the related equation $p^a \pm q^b \pm r^c \pm s^d = 0$ has only finitely many solutions when p, q, r and s are distinct primes. However, their methods do not seem to apply when the terms in the equation are not powers of distinct primes.

The authors and J.L. Brenner [1], [2], [4] have recently developed

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