ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 13, Number 2, Spring 1983

SOME RATIONAL CONTINUA

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In this note there are presented some examples of rational continua. The first example is of a rational continuum X (of rim-type 2) and a confluent mapping of X onto a non-rational continuum. This answers in the negative Problem III which was posed by A. Lelek in [**6**, p. 57]. In the second example there is presented a rational continuum X of rim-type 2 and a confluent mapping of X onto a rational continuum of rim-type 3. These two examples give negative answers to the following question which was posed by B.B. Epps in his dissertation [**3**, p. 6]: If X is a rational continuum of finite rim-type and f: $X \to Y$ is a confluent map, is the rim-type of Y less than or equal to the rim-type of X? In the second example there is given a rational, uniquely arcwise connected continuum X which contains a dense ray (continuous one-to-one image of [0, 1)) which is of first category in X. This answers in the negative a question posed by J.B. Fugate in a talk given at the Auburn Topology Conference in March 1976 (see [**4**, Question 2]). The third and final example in this note is of a hereditarily locally connected continuum X which contains a dense ray which is of first category in X.

I wish to thank Professors A. Lelek and J.R. Martin for several very helpful conversations.

1. Definitions and preliminaries. Our notation follows that of Whyburn [**9**]. By a *continuum* is meant a compact, connected, metric space. The set of natural numbers is denote by **N**. A continuum X is *rational at a point* $x \in X$ if X has a neighbourhood basis at x of open sets with countable boundaries. A continuum is *rational* if it is rational at each of its points. A sequence of sets is said to form a *null sequence* if the diameters of the sets converge to zero. A continuous function f of a continuum X onto a continuum Y is *confluent* if for each continuum C in Y each component of $f^{-1}(C)$ maps onto C. Let Cl(A) and Bd(A) denote the closure and boundary, respectively, of a set A. By a *neighbourhood* we shall mean an open neighbourhood.

If A is a subset of a space X, let A' denote the derived set of A. Let

This work was supported in part by National Research Council (Canada) grant No. A5616.

Received by the editors on August 13, 1976.