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LOCALLY FINITE GROUPS WHOSE IRREDUCIBLE MODULES ARE FINITE DIMENSIONAL

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1. Introduction. Let G be a group and k be a (commutative) field. The problem of determining when every irreducible kG-module has finite dimension over k has recently been considered by Snider [12], Musson [9] and Wehrfritz [13] [14] [15] for groups which are close to being soluble. In particular, [15] deals with the case when k has characteristic $p > 0$ and G belongs to a certain class of locally finite generalized soluble groups. This has prompted publication of the following result, which is a stronger version of [15] Theorem 1. The terminology is as follows. Let R be a ring with 1, V an irreducible right R-module, and $E = \mathrm{End}_R V$ be the endomorphism ring of V. Then E is a division ring, by Schur's Lemma, and we say that V has *finite endomorphism dimension* if $\dim_E V < \infty$.

THEOREM. *Let G be a locally finite group and k be a field of characteristic $p > 0$. Then every irreducible kG-module has finite endomorphism dimension if and only if $G/0_p(G)$ is almost abelian.*

We recall that a group *almost* has a certain property if it has a subgroup of finite index with the property.

This theorem was proved in 1975 as the result of a stimulating discussion with R.L. Snider. The case when G has no elements of order p had previously been dealt with by Farkas and Snider [1] (see also [3] Theorem B), and this will play an important part in the proof of the present theorem.

If k and G are as given and V is an irreducible kG-module, then as the augmentation ideal of $0_p(G)$ is nil, $0_p(G)$ operates trivially on V. Thus we can view V as an irreducible $G/0_p(G)$-module, and if this group is almost abelian, then V has finite endomorphism dimension (see [3] Lemma 1.1 for example). Thus only the necessity of the condition is at issue in the theorem. A key fact, which is very helpful in working with fields of positive characteristic, is the following, which the author learned from a preliminary version of [1].

LEMMA 1.1. *Let G be a locally finite group, k be a field of positive characteristic, V be an irreducible (right) kG-module, and $E = \mathrm{End}_{kG} V$. Assume that $\dim_E V < \infty$. Then E is a (commutative) field.*

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