ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 13, Number 2, Spring 1983

LOCAL UNIFORM APPROXIMATION BY FUNCTIONS IN A UNIFORM ALGEBRA

DAVID M. WELLS

Throughout this paper X will denote a compact Hausdorff space and A will be a function algebra on X. A is *local* if A contains each function $f \in C(X)$ for which there is a collection of closed subsets $\{K_1, \ldots, K_k\}$ of X whose interiors cover X, with $f|_{K_i} \in A|_{K_i}$ for each $i = 1, \ldots, k$. We will call A *strongly local* if the requirement $f|_{K_i} \in A|_{K_i}$ in the above definition can be weakened to $f|_{K_i} \in A_{K_i}$. If A is strongly local it has the following property, not shared by other local function algebras: uniform approximability of a function in $C(X)$ by functions in any dense subalgebra of A is implied by local approximability. Our main result is Theorem 2, which implies that a function algebra with a certain separation property is necessarily strongly local.

Strongly local function algebras are mentioned briefly in [2], where they are referred to as "approximately local". It is known that $R(X)$, the uniform closure in $C(X)$ of the rational functions with no poles on X, is strongly local for any compact plane set X. It is easily shown that $P(X)$, the uniform closure of the polynomials in $C(X)$, is not strongly local for a plane set X which is not polynomially convex. In fact if $x \in X$, λ is in a bounded component of $C \backslash X$ and $f(z) = 1/(z - \lambda)$, then $f \notin P(X)$ but $f|_K \in A_K$ if K is the intersection of X with a sufficiently small closed disc centered at x.

We will be concerned with algebras possessing separation properties. A is *approximately normal* if for any pair of disjoint closed subsets K_1 and K_2 of X and any $\varepsilon > 0$, there is a function $h \in A$ for which $\|1 - h\|_{K_1} < \varepsilon$ and $\|h\|_{K_2} < \varepsilon$. It was shown in [5] that an approximately normal algebra defined on an interval must be local. Approximately normal function algebras in general need not be strongly local. For example, the disc algebra defined on the circle is well-known to be approximately normal, but as a consequence of the preceding paragraph, it is not strongly local. However, approximately normal function algebras on an interval can be shown to possess an intermediate property. We will call A *boundedly strongly local* if A contains each function $f \in C(X)$ for which there is a collection $\{K_1, \ldots, K_k\}$ of closed subsets of X whose interiors

Received by the editors on June 23, 1981.