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ANALOGOUS FUNCTION THEORIES FOR THE HEAT, WAVE, AND LAPLACE EQUATIONS

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ABSTRACT. Transmutation operators are used to establish analogous function theories for standard and radial versions of the heat, wave, and Laplace equation. Under these transformations correspondences are established relating fundamental solutions, polynomial solutions, associated functions, generating functions, Fourier transform criteria, and expansion theorems. In some cases, the transmutation operators must be interpreted in the generalized sense as acting on distributions.

1. Introduction. In 1966, D.V. Widder [11] pointed out numerous analogies between classical function theory and representation theory for solutions of the heat equation. He did this by comparing a table of properties for representations of analytic functions to a corresponding table of properties for representations of heat functions. This work did not directly connect the results in the two tables by means of constructive isomorphisms. One of the purposes of this paper is to indicate how the results in related partial differential equations can be used to accomplish this.

In a series of papers, [4–9] the authors have shown how various elliptic, parabolic, and hyperbolic partial differential equations can be related through transmutation operators. In particular, [8] we have shown that various polynomial solutions of elliptic and hyperbolic equations can be obtained from the heat polynomials and radial heat polynomials, and how these in turn can be used to represent solutions of problems involving these equations. In many cases, the transmutation operators do not exist in the classical sense and therefore must be interpreted in a generalized sense as acting on distributions. Once this is done, it is possible to show that classical analytic function theory can be obtained directly from the representation theory for heat functions. In this case, representation of heat functions in a strip in terms of heat polynomials [10] corresponds to

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