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# SELF-REPRODUCING KERNELS AND BILINEAR FORMULAS FOR Q-ORTHOGONAL POLYNOMIALS

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**1.** Recently Ismail [7] obtained the connection relations and bilinear formulas for the Jacobi and Hahn polynomials by using fractional operators. Later on, Rahman [9, 10] proved some related results by using a completely different approach. Al-Salam and Ismail [1] obtained reproducing kernels for $q$-Jacobi polynomials which are $q$-analogues of Ismail's results [7].

In §3 of this paper, following Rahman [10] we prove the following formula for $q$-Jacobi polynomials

$$\begin{aligned}&\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{[q^c;q]_{m+n}[q^d;q]_{m+n}[xq^{1+b};q]_n[yq^{1+b};q]_n(xy)^m z^{m+n}}{[q;q]_m[q;q]_n[q^{1+a};q]_m[q^{1+b};q]_n q^{(1+b)n}}\\ (1.1)\qquad &=\sum_{r=0}^{\infty}\frac{[q^{1+a};q]_r[q^{1+a+b};q]_r[q^c;q]_r[q^d;q]_r\, z^r q^{r(r-1)}}{[q;q]_r[q^{1+b};q]_r[q^{1+a+b};q]_{2r}}\\ &\cdot {}_2\phi_1\left[\begin{matrix}q^{c+r},\ q^{d+r};\ q:\ zq^{-1-b}\\ q^{2+a+b+2r}\end{matrix}\right]P_r(x;a,b;q)\,P_r(y;a,b;q),\end{aligned}$$

a $q$-analogue of a result of Feldheim [4].

In §4 we derive some self reproducing kernels and bilinear sums for the $q$-Hahn polynomials which are $q$-analogues of results of Rahman [9]. In §5 we also obtain $q$-analogues of Ismail's [7] connection relations and bilinear formulae for Hahn polynomials.

§6 contains an extension of a result of Andrews and Askey [2] (for $q$-Jacobi polynomials) to $q$-Racah polynomials from which we obtain an interesting bilinear formula for $q$-Racah polynomials.

The formulae obtained in this paper remain valid if we replace $q^a$, $q^b$, $\ldots$, by $a, b, \ldots$ respectively.

**2. Definitions and notations.** Let $|q| < 1$, $[q^a;q]_n = (1-q^a)(1-q^{a+1})\ldots(1-q^{a+n-1})$, $[q^a;q]_0 = 1$, $[q^a;q]_\infty = \prod_{j=0}^{\infty}(1-q^{a+j})$ and the generalized basic hypergeometric series is defined as

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