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INFORMATION REGULARITY AND THE CENTRAL LIMIT QUESTION

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ABSTRACT. Two strictly stationary sequences $(X_k, k = \ldots, -1, 0, 1, \ldots)$ of random variables are constructed for which the "information regularity" condition is satisfied, the second moments are finite, and Var $(X_1 + \cdots + X_n) \to \infty$ as $n \to \infty$, but the central limit theorem fails to hold. In the first, the mixing condition based on "maximal correlations" is also satisfied. In the second, the mixing rate for "information regularity" is permitted to be arbitrarily fast (but m-dependence is not permitted) and n^{-2} Var$(X_1 + \cdots + X_n)$ is permitted to approach 0 arbitrarily slowly. In the first sequence there is partial attraction of $(X_1 + \cdots + X_n)$ to some mixtures of mean-zero normal distributions, and in the second there is partial attraction to all infinitely divisible laws.

Throughout this article, if A is a Borel subset of the real number line $\mathbf{R}$, then a "probability measure on A" will mean a probability measure on the measurable space $(A, \mathscr{B}_A)$ where $\mathscr{B}_A$ is the class of Borel subsets of A. The symbol * will denote convolution, applied to probability measures and probability distribution functions on $\mathbf{R}$ (or on an appropriate Borel subset A of $\mathbf{R}$). The indicator function of a set D will be denoted 1_D. To avoid subscripts of subscripts, we will often write such terms as a_b in the form $a(b)$. Similarly, if we mention a measure $\mu(n)$ we mean $\mu(n)(\cdot)$; the n is like a subscript, not an argument.

Let $(\Omega, \mathscr{F}, P)$ be a probability space. For any collection Y of random variables let $\mathscr{B}(Y)$ denote the Borel σ-field generated by Y. A "proper partition" of Ω will mean a partition of Ω into a finite set of events $\{A_1, \ldots, A_N\} \subset \mathscr{F}$ such that $P(A_n) > 0$, $\forall n$.

For any two σ-fields $\mathscr{A}$ and $\mathscr{B}$ define the following measures of dependence:

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