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A BANG-BANG RESULT FOR AN UNDAMPED SECOND-ORDER EVOLUTION EQUATION OF SOBOLEV TYPE

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ABSTRACT. We determine bang-bang properties of an optimal control for an undamped second-order evolution equation of Sobolev type. The property is shown to depend on the point of control. However, smoothing of the Sobolev equation allows consideration of domains in $\mathbf{R}^p$ for $p = 1, 2$, or 3. A time optimal and a fixed-time problem are considered.

1. Let Ω be a nonempty open bounded subset of $\mathbf{R}^p$ with $p = 1, 2$, or 3 with smooth boundary Γ and let $Q = \Omega \times (0, T)$ with lateral boundary denoted by $\Sigma = \Gamma \times (0, T)$. In this paper we study a control problem with an underlying equation given by the Sobolev equation

$$(1)\qquad \begin{aligned} (1 - \lambda\Delta)y_{tt} - \Delta y &= v(t)\delta(x - a) \text{ in } Q,\\ y(x, t) &= 0 \text{ on } \Sigma,\\ y(x, 0) = y_t(x, 0) &= 0 \text{ in } \Omega \end{aligned}$$

and with the optimization problem

$$(2)\qquad \begin{aligned} &\text{minimize} \int_\Omega (y(x, T; v) - z(x))^2 dx\\ &\text{subject to } v \in U_{\text{ad}} \end{aligned}$$

where

$$U_{\text{ad}} = \{v \in L^\infty(0, T): \|v\|_{L^\infty(0,T)} \leqq 1\}.$$

We assume that $a \in \Omega$, $z \in L^2(\Omega)$, and $\lambda = 1$. For ease in our discussion we take $\Omega = (0, 1)$ and point out extensions to higher dimensions.

Problems such as (1) arise in the study of longitudinal vibrations in a beam, see Love [7], and [2, 3, 4, 9]. In these studies the presence of the

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