

PIECEWISE LINEAR APPROXIMATE FIBRATIONS

R. E. GOAD

ABSTRACT. A pl map is an approximate fibration if and only if all the maps of any iterated mapping cylinder decomposition are homotopy equivalences. This leads to a classifying space.

In this paper we provide a local characterization for piecewise linear maps which are also approximate fibrations. We use the approach taken by A. E. Hatcher in [5] to apply his higher simple homotopy theory to the classification of PL fibrations. This leads to a classifying space for PL approximate fibrations. All of the proofs are elementary but, hopefully, the results have sufficient intrinsic interest to justify themselves.

I. Local characterization. We begin with a few basic definitions and assumptions. All metric spaces will be equipped with a fixed metric, denoted by $d(a, b)$. In the case of the unit interval, $I = [0, 1]$, the metric is the usual absolute value of $a - b$.

DEFINITION I.1. (See [1].) A proper surjection $p: E \rightarrow B$ of metric spaces is an approximate fibration if, for every metric space D , every lifting problem

$$\begin{array}{ccccc}
 & & & & E \\
 & & & \nearrow^{H_0} & \downarrow p \\
 D \times \{0\} & \xrightarrow{i} & D \times I & \xrightarrow{h} & B \xrightarrow{\varepsilon} (0, \infty)
 \end{array}$$

has an approximate solution

$$\begin{array}{ccccc}
 & & & & E \\
 & & & \nearrow^H & \downarrow p \\
 D \times \{0\} & \xrightarrow{i} & D \times I & \xrightarrow{h} & B \xrightarrow{\varepsilon} (0, \infty)
 \end{array}$$

AMS 1970 subject classifications: Primary 55F65, 57C99, 55F15.

Key words and phrases: Approximate fibration, iterated mapping cylinder, classifying space.

Received by the editors on April 11, 1980.