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DIFFERENTIAL-BOUNDARY OPERATORS AND ASSOCIATED NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS

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1. Introduction. It has been known for some time that differential-boundary operators play an important role in the adjoint theory of linear differential operators with general boundary conditions. In addition to the classical work of Feller [10] and Phillips [21], the theory of differential-boundary operators has been applied to such diverse fields as spline analysis (Brown [2], Brown and Krall [3]), variational and oscillation theory (Reid [22, 24]), boundary control of parabolic (Seidman [27]) and hyperbolic (Russell [25, 26]) partial differential equations. However, it is interesting to note that although the early work of Feller and Phillips was concerned with the well-posedness of Cauchy problems associated with these operators most of the current literature on differential-boundary operators does not consider this problem. Since 1960 the theory has generally been devoted to the study of adjoint operators, derivation of Green's matrices and eigenfunction expansions (see [14–19] and the survey paper by Krall [20]). In this paper we study a general class of 1st order differential-boundary operators and derive necessary conditions and sufficient conditions for these operators to generate C_0-semigroups. Moreover, we show that there exists a fundamental relationship between these operators and Cauchy problems for neutral functional differential equations. Although we shall not pursue the point, similar observations have recently been used to develop numerical methods for approximating solutions and optimal controls for certain integro-differential systems (see [7]).

Notation used in the paper is fairly standard. For example, $L_2 = L_2([0, 1]; \mathbf{R}^n)$ denotes the usual Lebesgue space of $\mathbf{R}^n$-valued "functions" on [0, 1] whose components are square integrable. We shall also make use of the Sobolev space $H^1 = H^1([0, 1]; \mathbf{R}^n)$ and the Banach space of

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