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STEFFENSEN TYPE INEQUALITIES

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Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

1. Introduction. Stephensen's inequality has a long and varied history, see Mitrinović [3, pg. 107–119], for example. The simplest version is the following theorem.

THEOREM A. *Let F be non-decreasing and* $0 \leqq g \leqq 1$, *both functions continuous. Then*

$$\int_0^a f\,dx \leqq \int_0^1 fg\,dx \leqq \int_{1-a}^1 f\,dx \tag{1}$$

where $a = \int_0^1 g\,dx$.

Recently Milovanović and Pečarič [2] have shown that the same conclusions hold if $0 \leqq g \leqq 1$ is replaced by

$$\text{(i)} \qquad \int_x^1 g\,dt \geqq 0 \text{ and } \int_0^x g\,dt \leqq x,\ x \in [0, 1];$$

for the left hand inequality of (1) and for the right hand inequality

$$\text{(ii)} \qquad \int_x^1 g\,dt < 1-x,\ \int_0^x g\,dt \geqq 0,\ x \in [0, 1].$$

They further prove versions of (1) with f satisfying a higher monotonicity.

In this paper we show that Theorem A as well as the versions of Theorem A proved in [2] are simple corollaries of Theorem D and its extensions proved in this paper.

THEOREM B. *Let* M_0 *be the class of non-negative non-decreasing integrable functions, and* μ *a (signed) regular Borel measure. Then*

$$\int_0^1 f\,d\mu \geqq 0 \tag{2}$$

holds for all $f \in M_0$ *if and only if*

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