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PREDATOR INFLUENCE ON THE GROWTH OF A POPULATION WITH THREE GENOTYPES II

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Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

1. Introduction. The system of ordinary differential equations

$$
\begin{aligned}
x_1' &= (x_1 + (1/2)x_2)^2/x^2)\, B(x) - (\Delta(x) + yP_1(x, y))(x_1/x)\\
x_2' &= (2(x_1 + (1/2)x_2)(x_3 + (1/2)x_2)/x^2)B(x)\\
&\qquad - (\Delta(x) + yP_2(x, y)\, (x_2/x)\\
x_3' &= ((x_3 + (1/2)x_2)^2/x^2)\, B(x) - (\Delta(x) + yP_3(x, y))(x_3/x)\\
y' &= y(-s + k \sum_{i=1}^{3} P_i(x, y))\\
x_i(0) &= x_{i0} > 0, y(0) = y_0 > 0, x = x_1 + x_2 + x_3
\end{aligned} \tag{1.1}
$$

was investigated in [11] as a model of a predator, denoted by y, feeding on a prey, denoted by x, which consists of three genotypes, denoted by x_1, x_2, x_3, corresponding to a one locus, two allele, genetic model. Without the predator, this system of equations also appears in [1] and [5]. In the genetics literature these three genotypes are frequently denoted by AA, Aa, aa, emphasizing the two choices for each allele at the distinguished location. If one of the genetic characteristics is recessive, the organism will appear as two varieties, called phenotypes, and the resulting difference, say color [3], may affect the susceptibility of the organism to predation.

Standard hypotheses to model the predator-prey relationship (intermediate type models in the language of [9]) are:

(H-1) $\Delta(x) \geqq 0,\ B(0) = \Delta(0) = 0,\ B'(0) > \Delta'(0) \geqq 0.$
If $y > 0,\ P_i(x, y) = 0 \Leftrightarrow x = 0,\ P_{ix}(x, y) > 0.$

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