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A DISFOCALITY FUNCTION FOR A NONLINEAR ORDINARY DIFFERENTIAL EQUATION

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Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

We will be concerned with the differential equation

$$(1) \qquad y^{(n)} = f(x, y, \ldots, y^{(n-1)})$$

where we will make some or all of the assumptions:

(A) f is continuous on $J \times \mathbf{R}^n$ (J a subinterval of the reals, $\mathbf{R}$).

(B) solutions of initial value problems (IVP's) are unique and exist on the whole interval J.

(C) if $\{y_n\}$ is a sequence of solutions which is uniformly bounded on a nondegenerate compact interval $[c, d] \subset J$, then there exists a subsequence $\{y_{n_k}\}$ such that each of the sequences $\{y_{n_k}^{(i)}\}$, $i = 0, \ldots, n-1$, converges uniformly on compact subintervals of J.

(D) $f_i(x, y, \ldots, y^{(n-1)}) = (\partial/\partial y^i) f(x, y, \ldots, y^{(n-1)})$, $i = 0, \ldots, n-1$ is continuous on $J \times \mathbf{R}^n$.

For information concerning the compactness condition (C) see [6] and the references given there.

We now introduce much of the same notation used by Muldowney [9]. Let $\tau = (t_1, \ldots, t_n)$. We say that $y(x)$ has n zeros at τ provided $y(t_i) = 0$, $1 \leqq i \leqq n$, and $y(t_i) = y'(t_i) = \cdots = y^{(m-1)}(t_i) = 0$ if a point t_i occurs m times in τ. A partition $(\tau_1; \ldots; \tau_\ell)$ of the ordered n-tuple $(t_1, \ldots, t_n)$ is obtained by inserting ℓ-1 semicolons in the expression. Let $m_i = |\tau_i|$ be the number of components of τ_i (so $\sum_{i=1}^{\ell} m_i = n$). We allow $m_i = 0$ (in which case we might think of τ_i as being a zero tuple or the empty set). We say that $(\tau_1; \ldots; \tau_\ell)$ is an increasing partition of $(t_1, \ldots, t_n)$ provided $t_1 \leqq t_2 \leqq \cdots \leqq t_n$ and if t is a component of τ_i and s is a component of τ_j with $i < j$ then either $t < s$ or $t = s$ and $i + m \leqq j$ where m is the multiplicity of t in τ_i.

We say that (1) is right $(m_1; \ldots; m_\ell)$-disfocal on J, $m_1 + \cdots + m_\ell = n$, $0 \leqq m_j \leqq n - j + 1$, provided there do not exist distinct solutions of

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