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A NOTE ON MONOTONICITY PROPERTIES OF A FREE BOUNDARY PROBLEM FOR AN ORDINARY DIFFERENTIAL EQUATION

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Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

Introduction. A number of results have appeared in which the method of lines (also known as Rothe's method) is used to construct approximate solutions to two dimensional parabolic free boundary problems. This method replaces the partial differential equation with a sequence of ordinary differential equations by substituting a difference term for the time derivatives. For example, in a paper by Sackett [5], the solution to a problem for the heat equation is reduced to a sequence of problems for $u_n(x) \cong u(x, nh)$ of the form

$$u'' = h^{-1}(u - k_n(x)),\ x \in (0, s_n);$$
$$u(0) = f_{1n},$$
$$u(s_n) = f_{2n},\ u'(s_n) = g_n,$$
$$u_n(x) := u(x).$$

where f_{1n}, f_{2n}, and g_n are given from the boundary conditions of the two dimensional problem and $k_n(x)$ is a suitably extended solution $u_{n-1}(x)$ of the previous problem in the sequence. In [4], Meyer considers a problem which produces a similar sequence of ordinary differential equations, but with the boundary conditions

$$u'(0) = \alpha_n,$$
$$u(s_n) = u'(s_n) = 0.$$

In both of these results, monotonicity properties of the boundary value problems with respect to the boundary conditions and the functions $k_n(x)$ are used to obtain bounds on the solutions and on the location of the free boundary.

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