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COMPARISON TECHNIQUES AND THE METHOD OF LINES FOR A PARABOLIC FUNCTIONAL EQUATION

J. BEBERNES AND R. ELY

Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

1. Introduction. In a recent paper [3], a detailed mathematical analysis for the implicit integro-differential equation

$$\text{(I)} \qquad \theta_t - \Delta\theta = \delta e^{\theta} + ((\gamma - 1)/\gamma)(1/\text{vol } \Omega) \int_{\Omega} \theta_t dy$$

was given. Equation (I) is the model for the induction period for the thermal explosion process of a compressible reactive gas in a bounded container.

In particular in [3], it was shown that the solution of (I) is always dominated by the solution of the explicit integro-differential equation

$$\text{(E)} \qquad u_t - \Delta u = \delta e^{u} + ((\gamma - 1)/\text{vol } \Omega)\, \delta \int_{\Omega} e^{u} dy$$

on their common interval of existence, if $\Omega = \mathscr{B}$, a ball in $\mathbf{R}^n$.

The purpose of this paper is to analyse initial-boundary value problems for a class of explicit integro-differential equations (see IBVP (1)–(2)) which include (E) (see IBVP (13)–(14)) as a special case.

2. Known existence results. Consider the scalar integro-partial differential equation

$$\text{(1)} \qquad u_t - \Delta u = f(t, u) + \int_{\Omega} g(t, u) dx$$

with the initial-boundary conditions

$$\text{(2)} \qquad \begin{aligned} u(x, t) &= u_0(x), && (x, t) \in \Omega \times \{0\}, \\ u(x, t) &= 0, && (x, t) \in \partial\Omega \times [0, \infty), \end{aligned}$$

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