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BOUNDARY VALUE PROBLEMS FOR PAIRS OF SECOND ORDER EQUATIONS CONTAINING A SMALL PARAMETER

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Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

1. Introduction. In this paper we study weakly coupled second order systems

$$\begin{aligned} \varepsilon x'' &= F(t, x, x', y), \\ y'' &= G(t, x, y, y'), \end{aligned} \tag{1}$$

where ε is a small positive parameter, with boundary conditions

$$x(0) = A,\ x(1) = B,\ y(0) = C,\ y(1) = D. \tag{2}$$

We associate with boundary value problems of type (1), (2) certain "reduced" problems

$$\begin{aligned} 0 &= F(t, x, x', y), \\ y'' &= G(t, x, y, y'), \end{aligned} \tag{3}$$

with a proper subset of the boundary conditions (2). Such problems have been previously examined by Hoppensteadt [4], Vasil'eva and Butusov [11], Fife [1] and others. These authors have demonstrated that solutions to (1), (2) exist under certain conditions and exhibit boundary layer behavior. Also, the occurence of internal layers for certain autonomous problems has been studied by Fife [1] for boundary conditions of type (2) and by Mimura, Tabata and Hosono [10] for boundary conditions of Neumann type. However, for a given problem, the conditions imposed by the above authors are difficult to check.

The purpose of this paper is to present explicit conditions on (1) so that for each "stable" solution of the reduced problem (3), there is a solution of (1), (2) which is approximated by the solution of the reduced problem for small $\varepsilon > 0$ outside of boundary layers at one or both endpoints of [0, 1]. Since our main technique is comparison with linear

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