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NONUNIFORM NONRESONANCE CONDITIONS AT THE TWO FIRST EIGENVALUES FOR PERIODIC SOLUTIONS OF FORCED LIENARD AND DUFFING EQUATIONS

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Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

1. Introduction. This paper is devoted to the study of the periodic boundary value problem for the Lienard differential equation

$$x''(t) + f(x(t))x'(t) + g(t, x(t)) = 0 \tag{1.1}$$

and its special case, the Duffing equation

$$x''(t) + cx'(t) + g(t, x(t)) = 0. \tag{1.2}$$

Without loss of generality, we can assume that the boundary conditions are taken on the interval $[0, 2\pi]$, namely,

$$x(0) - x(2\pi) = x'(0) - x'(2\pi) = 0. \tag{1.3}$$

Our results are only significant for the so-called forced case, i.e., when $g(t, 0) \not\equiv 0$.

There is a vast literature dealing with problems (1.1–1.3) and (1.2–1.3) and we refer to **[10]** and its bibliography for further references. For f continuous and g of the form $g(t, x) = h(x) - e(t)$ with h and e continuous and e 2π-periodic, Reissig **[9]** has proved that problem (1.1–1.3) has at least one solution if

$$0 < \liminf_{|x|\to\infty} \frac{h(x)}{x} \leqq \limsup_{|x|\to\infty} \frac{h(x)}{x} < 1.$$

On the other hand, Amaral and Pera **[1]** have proved that problem (1.2–1.3) has at least one solution for the case where $c = 0$, g is 2π-periodic in t and continuous,

$$\alpha \leqq \gamma_-(t) = \liminf_{|x|\to\infty} \frac{g(t, x)}{x} \leqq \limsup_{|x|\to\infty} \frac{g(t, x)}{x} = \gamma_+(t) \leqq \beta < 1$$

uniformly in $t \in [0, 2\pi]$, for some $\alpha \in \mathbf{R}$, and $\int_0^{2\pi} \gamma_-(t)dt > 0$.

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