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NONOSCILLATION RESULTS FOR SECOND ORDER NONLINEAR DIFFERENTIAL EQUATIONS

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Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

ABSTRACT. By means of a change of variable along with appropriate energy functions, criteria are obtained which guarantee that all solutions of the second order nonlinear equation $y'' + p(x)y^\gamma = 0$, $p > 0$, $\gamma > 1$, are nonoscillatory. These results strengthen known nonoscillation criteria.

1. Introduction. Consider the nonlinear second order equation

$$y'' + p(x)y^\gamma = 0 \tag{1.1}$$

where $p > 0$ is locally integrable on $[a, \infty)$ and $\gamma > 1$ is the quotient of odd positive integers. We shall be interested in obtaining criteria for all nontrivial solutions of (1.1) to be nonoscillatory (i.e., have only finitely many zeros).

It was shown by Atkinson [1] that all solutions are oscillatory if and only if $\int_a^\infty xp(x)\,dx = \infty$. We refer to the survey papers [14], [15] and [9] for detailed bibliographies. In contrast to the linear case $\gamma = 1$, however, equation (1.1) permits the coexistence of both oscillatory and nonoscillatory solutions. Thus, while $\int_a^\infty xp(x)\,dx < \infty$ guarantees the existence of at least one nontrivial nonoscillatory solution, it remains of interest to find criteria for the existence of an oscillatory solution to (1.1) or conditions which imply that all solutions are nonoscillatory. These have been studied less frequently. For the former we refer to [5], [6], [7] and [8]. Nonoscillation criteria may be found in [4], [12] and [13].

Our technique is to employ a suitable change of variables along with an appropriate energy function. The results obtained strengthen known criteria. For example, Nehari [13] has shown that if $p(x)(x \log x)^{(\gamma+3)/2}$ is nonincreasing on $[a, \infty)$, then equation (1.1) is nonoscillatory. This was subsequently improved by Chiou [4] who showed that if $p(x)x^{(\gamma+3)/2}(\log x)^\beta$ is nonincreasing on $[a, \infty)$, where $\beta > (\gamma + 1)/4 - 1/(\gamma + 1)$,

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