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NORMAL FIBRATIONS AND THE EXISTENCE OF TUBULAR NEIGHBORHOODS

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ABSTRACT. To each pair (M, N) of Hilbert cube manifolds for which N is locally flat of codimension n in M, there corresponds a normal Hurewicz fibration over N whose fibers have the homotopy type of S^{n-1}. It is shown that N has a closed tubular neighborhood in M if and only if the normal fibration is fiber homotopically equivalent to some abstract S^{n-1} – bundle over N.

1. Introduction. A closed subspace N of a manifold M is *locally flat* (*with codimension n*) if for each $x_0 \in N$ there is an open neighborhood U of x_0 in N and an open embedding $h: U \times \mathbf{R}^n \to M$ such that $h(x, 0) = x$ for all $x \in U$, where $\mathbf{R}^n$ is n-dimensional Euclidean space. The pair (M, N) is then called a *locally flat pair*.

It is of particular interest to determine, if (M, N) is a locally flat pair, whether N has a tubular neighborhood in M. A *tubular neighborhood* is a neighborhood E of N in M for which there exists a retraction $p: E \to N$ such that (E, p, N) is a locally trivial fiber bundle with 0-section N and fiber F which is either $\mathbf{R}^n$ or the Euclidean n-ball B^n. If E is open in M and $F = \mathbf{R}^n$, E is called an *open tube*; likewise, if E is closed and $F = B^n$, E is called a *closed tube*. The *boundary* ∂E of a closed tube is the combinatorial boundary of E, which is an S^{n-1}-bundle. A locally flat pair of topological (as opposed to differentiable) manifolds need not admit a tubular neighborhood (see, for example, **[13]**).

The subject of this paper is the tubular neighborhood question for Hilbert cube manifolds. A *Q-manifold* is a separable metric space with a basis consisting of elements homeomorphic to open subsets of the Hilbert cube Q. Equivalently, the basis elements may be required to be homeomorphic to $Q \times [0, 1)$ **[1]**, a fact which will be used repeatedly. Henceforth, (M, N) will always be used to mean a locally flat Q-manifold pair with codimension n.

It is shown in **[11]** that if (M, N) has codimension **2**, then N always has a closed (and thus also an open) tubular neighborhood, a result analogous to the finite dimensional result of Kirby and Siebenmann **[10]**. (For a

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