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ON FACTOR STATES

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1. Introduction. Let **A** be a (complex) C^*-algebra, f a state on **A**. Let $\{\pi_f, H_f, x_f\}$ denote the *GNS* triple corresponding to f. f is a *factor state* if $\pi_f(\mathbf{A})''$ is a factor ($'$ denotes taking the commutant). Traditionally, the set of factor states has played an important role in the integration and disintegration theory of states and representation of C^*-algebras ([**6**], Chapter 8; [**16**], Chapter 3). Recently, in work of Choi-Effros and Connes characterizing separable nuclear C^*-algebras ([**3**], [**4**], [**5**]); see also [**7**]) and work of Anderson and Bunce on the Stone-Weierstrass problem ([**1**]), the set of factor states has also been important. The detailed study of factor states was started by Kadison in [**11**] (see Theorem A, p. 306). In view of the above recent work, it therefore seems worthwhile to continue this study, and this paper is thus a contribution in that direction.

In §2 of the present paper, we give a characterization of factor states analogous to the Segal characterization of pure states, and use this to obtain an extension theorem for factor states. In §3 we characterize commutative and elementary C^*-algebras by a condition on their set of factor states, in a way which exhibits these two classes of C^*-algebras as opposite extremes of a common phenomenon. In Section 4 we present a vector state characterization of irreducibility of C^*-algebras which grew out of the investigations of the two preceding sections.

We review and fix our notation for the sequel. If **A** is a C^*-algebra, f a state on **A**, π_f, H_f, and x_f will denote respectively the representation, Hilbert space, and unit cyclic vector arising in the *GNS* construction corresponding to f. $\mathbf{A}_+$ and $\mathbf{A}_+^*$ denote respectively the positive elements of **A** and the positive linear functionals on **A**. We denote unitary equivalence of algebras, representations, and operators by $\sim$. If X is a normed algebra or linear space and α is a cardinal number, $\alpha \cdot X$ denotes the α-fold multiple of X with the standard algebra or normed linear space operations inherited from X. If S is a subset of X, we set Ball $S = \{s \in S: \|s\| \leqq 1\}$ and $S_1 = \{s \in S: \|s\| = 1\}$. If H is a (complex) Hilbert space, $B(H)$ will denote the W^*-algebra of all bounded operators on H and $C(H)$ will denote the C^*-algebra of compact operators on H.

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