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DIFFERENTIATION ON THE DUAL OF A GROUP: AN INTRODUCTION

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Introduction. This paper is the first of a series wherein we investigate a theory of differentiation on the dual of a group together with its applications to and its interrelationships with other aspects of the theory of groups and their representations. One of the major objectives in the study of locally compact group G is the computation of $\hat{G}$, the set of all (unitary equivalence classes of) continuous, unitary, irreducible representations of G equipped with a natural topology, the Fell topology, cf. 3.4, 18.1 [**8**]. There are immediate problems; first of all it is usually very difficult to compute all the elements of $\hat{G}$. Secondly, $\hat{G}$ with its natural topology is rarely Hausdorff; and elementary examples yield spaces $\hat{G}$ in which not all points are even closed. Finally $\hat{G}$, in general, seems to lack any natural elementary algebraic structure. For these reasons the study of a "differentiable structure" or of any other structure of $\hat{G}$ in any generality is very difficult; to see any structure at all one usually is forced to investigate specific classes of groups—or, indeed, specific groups. Of course, a great deal of study directed at specific groups and specific classes of groups has been carried out with immense success over the last several decades; and we shall directly benefit from these studies.

The seeming intractibility of $\hat{G}$ leads us to consider other structures closely related to $\hat{G}$, namely, the space of all continuous, unitary representations of G, denoted $\mathrm{Rep}_u(G)$. This space can be refined even further to $P(G)_1$, the space of diagonal coefficients (of norm one) of the elements of $\mathrm{Rep}_u(G)$, i.e., $P(G)_1$ is the collection of all continuous functions of positive type (of norm one) on G. The space $P(G)$ is often called the space of positive definite functions. (There are strong indications that even larger spaces can be usefully employed in the study of G and $\hat{G}$, but we leave this possibility totally untouched for now.)

The space $P(G)_1$ is a convex, Hausdorff, topological semigroup. To recover $\hat{G}$ one needs "only" to find the extreme points of $P(G)_1$ and then perform a canonical construction. Thus a thorough understanding of

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