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GENERALIZED ALTERNATIVE AND MALCEV ALGEBRAS

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1. Introduction. As observed in [1], both alternative algebras and Malcev algebras satisfy the flexible law

$$(1) \qquad (x, y, x) = 0,$$

and

$$(2) \qquad (zx, x, y) = -x(z, y, x),$$

where the associator $(a, b, c) = (ab)c - a(bc)$. Algebras satisfying in addition to (1) and (2) the identity

$$(*) \qquad (xz, x, y) = -(z, y, x)x$$

were studied initially by Filippov [1], who showed that a prime algebra of this sort (with characteristic $\neq 2, 3$) must be either alternative, Malcev, or a Jordan nil-algebra of bounded index 3. In this paper we shall consider algebras (with characteristic $\neq 2, 3$) which satisfy only (1) and (2). (Note that algebras opposite to these satisfy instead (1) and (*).) We shall prove that in this variety nil-semisimple algebras are alternative, and that prime algebras are either alternative or nil of bounded index 3. We shall also establish for finite-dimensional algebras the standard Wedderburn principal theorem.

To begin with, there are some elementary consequences of (1) and (2) which need to be noted. We first set

$$\begin{aligned} T(w, x, y, z) = (wx, y, z) - (w, xy, z) + (w, x, yz) \\ - w(x, y, z) - (w, x, y)z. \end{aligned}$$

It can be verified by simply expanding the associators that in any algebra $T(w, x, y, z) = 0$. Also, the linearized form of (2) is

$$(2') \qquad (zx, w, y) + (zw, x, y) = -x(z, y, w) - w(z, y, x),$$

so that

$$F(z, x, w, y) = (zx, w, y) + (zw, x, y) + x(z, y, w) + w(z, y, x) = 0.$$

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