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A UNIVERSAL EXAMPLE OF A CORE-FREE PERMUTABLE SUBGROUP

FLETCHER GROSS AND T.R. BERGER

Introduction. Let H be a core-free permutable subgroup of the group G. This means that there is no non-identity normal subgroup of G contained in H and that $HK = KH$ for every subgroup K in G. (The term quasinormal has been used instead of permutable, but we feel that permutable, Stonehewer's word, is preferable since it is more descriptive.) In proving results about the structure of H, a reduction often is made to the special case when G is a finite p-group and $G = HC$ for some cyclic subgroup C. As examples of the sort of results obtainable in this way, we mention two: (1) H is residually finite nilpotent ([**1**] and [**8**]). (2) If n is any integer, then the set $\{x \in H | x^n = 1\}$ is a nilpotent subgroup of H and the class and derived length of this subgroup are bounded from above by functions of n ([**2**]; the best-possible bounds are given in [**3**]).

The study of the special case $G = HC$ with C cyclic and G a finite p-group has also led to the construction of counter-examples. Thus, although Itô and Szep [**6**] showed that H is nilpotent if G is finite, H need not be solvable if G is infinite. This follows from applying Theorem 3.3 of [**1**] to the finite groups constructed by Stonehewer in [**9**]. Stonehewer's groups all have the special structure referred to earlier. A study of Stonehewer's examples suggested that there might be a "universal" example. The main result of this paper then is the following.

THEOREM, *Let p be any prime and n a positive integer. Then there is a group $G = H\langle x\rangle$ such that*:

(i) *H is a core-free permutable subgroup of G and x has order p^n.*

(ii) *If $G^* = H^*\langle x^*\rangle$ where H^* is a core-free permutable subgroup of G^* and x^* has order p^n, then there is one and only one monomorphism ϕ of G^* into G such that $\phi(x^*) = x$ and $\phi(H^*) \leqq H$.*

The group G in this theorem is a finite p-group which will be constructed as a transitive permutation group with H being the stabilizer of a point. This procedure was suggested by Stonehewer's work although his groups are not the same as ours.

Originally, it was our intention to use the above theorem to try to prove

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